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Cherenkov X-ray radiation in the area of the water transparency window with broken azimuthal symmetry

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The features of Cherenkov and parametric radiation in the X-ray frequency range in the spectral range of the “Water Transparency Window” during the passage of relativistic particles through an isolated target and a regular stack of plates are analyzed and established. Numerical calculations of the output of polarized radiation have been performed for electrons with an energy of 10 MeV and a titanium target. The principal possibility of applying the effect to create innovative quasi-monochromatic directional sources of polarized X-ray radiation with high angular density is shown.

Keywords: cherenkov radiation, transient radiation, X-ray radiation, titanium, vanadium, water transparency window.

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Introduction

The study of phenomena related to Vavilov-Cherenkov radiation and transition radiation arising from the motion of accelerated relativistic particles through various materials has traditionally been a promising direction for the development of innovative detectors and radiation sources in various regions of the electromagnetic spectrum. Recent advances and prospects in this field are presented, in particular, in the recent review article [1].

As was shown in [2], the dielectric permittivity of certain substances near the edges of K- and L-absorption lines can exceed unity. This creates conditions for the occurrence of Cherenkov radiation in the X-ray frequency range. A theoretical analysis of the possibility of X-ray Cherenkov radiation in various materials was carried out in [3]. A discussion of the possible generation of X-ray Cherenkov radiation by ultrarelativistic electrons in carbon was first conducted in the experiments [4]. Paper [5]

reports the observation of X-ray Cherenkov radiation from electrons with an energy of 5 MeV in silicon near the L-absorption line edge at 99.7 eV. The authors of [6] detected the Cherenkov radiation from relativistic electrons with an energy $E = 10$ MeV in titanium and vanadium targets with a thickness of 10 μm at photon energies of 453 eV and 512 eV, respectively, at the normal incidence of particles on the target surface. The recorded photon yield per electron was 3.5×10^{-4} for the titanium target and 4.3×10^{-4} for the vanadium target, respectively. The region of the electromagnetic spectrum in the frequency range from 300 to 500 eV is of particular interest. Water is transparent in this spectral range (the so-called water window), but carbon and its organic compounds are not. The quasi-monochromatic Cherenkov radiation in this spectral region can be used to create X-ray microscopes for imaging biological tissue structures [7]. In this regard, the measurement results of [6] are of particular interest.

According to the data in [8], the maximum values of the real and imaginary parts of the dielectric permittivity in titanium are reached at a quantum energy of 454 eV, and the region where $\varepsilon' > 1$ is approximately 5 eV wide. At a quantum energy of 454 eV, $\varepsilon' = 1.007$ and $\varepsilon'' = 0.006$. It is easy to estimate that the threshold for the occurrence of the X-ray Cherenkov radiation with quantum energies near the L-absorption edge, for example in titanium, is reached a Lorentz factor $\gamma > 12$, which corresponds to an electron energy $E > 6$ MeV. The Cherenkov radiation angle relative to the electron velocity vector for electrons with the 10 MeV energy and a photon energy of 454 eV is $\theta_c \approx 3.78^\circ$. For relativistic electrons entering the target perpendicularly to the surface, the radiation is polarized in the plane of the wave vector and the normal to the surface (longitudinal polarization). For the oblique incidence of particles on the target, a radiation component polarized in the plane orthogonal to the radiation plane (transverse polarization) appears due to the breaking of azimuthal symmetry, and the angular distribution of radiation becomes asymmetric. The features of transition radiation and Cherenkov radiation for the oblique incidence of particles on the target were investigated in a number of monographs [9, 10], as well as in papers [11–17]. As shown in [11, 12], for oblique incidence, the intensity of the longitudinally polarized wave increases, while the width of the angular distribution decreases. In addition, azimuthal oscillations arise due to the interference of waves reflected from the front and rear surfaces of the target [13]. The method of polarization currents was used in [14] to study the spectral-angular distribution and polarization properties of the so-called X-ray "hybrid" radiation, i.e., under conditions where the interference between Cherenkov and transition radiation is significant.

Given below is an the analysis of Cherenkov radiation in the spectral region of the water transparency window for relativistic electrons entering a titanium target, as well as a regular stack of titanium plates under the conditions of the experiments cited above [6].

Spectral-Angular Distribution

One of the methods for solving the problem in this formulation is to match the normal and

tangential components of the fields at the interfaces. A general solution to such a problem is given in [10, 16]. However, it is possible to solve such problems without resorting to direct field matching. For this purpose, one can use the well-known result that the Fourier component of the field of a moving particle is equivalent to the field of summed stationary dipoles located along the particle trajectory. This approach was effectively used in [17] (the so-called "image method") for solving a number of problems on radiation from charged particles in a medium. Following that approach, we will represent the angular distribution of radiation in terms of direction cosines $\cos\theta_x$ and $\cos\theta_y$ (orthogonal axes x and y lie in the plane of the target surface, and the z -axis is directed along the normal to the surface). In contrast to the variables θ and φ (where θ and φ are the polar and azimuthal angles, respectively), this provides greater clarity. We assume that the front and rear surfaces of the target are parallel and have no roughness, and that the particle velocity is constant.

Analytical formulas for the spectral-angular density of radiation with longitudinal $dI^\parallel(\mathbf{k}, \omega) = I^\parallel(\mathbf{k}, \omega) d\omega d\Omega$ and transverse polarization $dI^\perp(\mathbf{k}, \omega) = I^\perp(\mathbf{k}, \omega) d\omega d\Omega$ in the frequency range $\omega, \omega + d\omega$ and the solid angle $d\Omega$ oriented in the direction of particle motion in a target of thickness L with a complex dielectric permittivity are given in [17] (formulas 18.25 and 18.26) and are rather cumbersome. However, more compact asymptotic formulas can be used for analyzing the radiation yield under the conditions of the above-cited experiments [6].

Indeed, such factors as the finite target thickness, a sufficiently large value of the imaginary part of the dielectric permittivity, and a relatively small difference of the real part from unity will lead to some features in the angular distribution of the resulting radiation when azimuthal symmetry is broken. First of all, we note that Cherenkov radiation is summed coherently along the entire trajectory in a transparent medium. When the imaginary part of the dielectric permittivity is equal to zero and the threshold condition is satisfied, this leads to the obvious divergence of the corresponding formulas. On the other hand, existence of absorption leads to the fact that the trajectory length, over which the radiation summation

occurs, becomes finite. This length can be easily estimated from relation $L_c \approx c / \omega \varepsilon$ (see, e.g., [10]), from which we obtain the estimate of $L_c / \lambda \approx 1 / \varepsilon'' \approx 166$ (λ is the radiation wavelength). Under the conditions of the experiments cited above, the thickness of the titanium target used is $10 \mu\text{m}$, so $L / \lambda \approx 3500$, which is much larger than length L_c (if the

wavelength is 0.28 nm , then $L_c \approx 0.5 \mu\text{m}$). Consequently, the wave reflected from the front surface in the direction of the rear surface will be strongly suppressed. Therefore, one can use the asymptotics of formulas (18.25) and (18.26) from [17] as $L / \lambda \rightarrow \infty$ for calculations of radiation intensity. We have the following for the spectral-angular density of radiation for the longitudinal polarization component:

$$I^{\parallel}(\mathbf{n}, \omega) = \frac{\beta_z^2 \cos^2 \theta_z |1 - \varepsilon|^2}{\left[(1 - \beta_x \cos \theta_x)^2 - \beta_z^2 \cos^2 \theta_z \right]^2 \sin^2 \theta_z} \times \left| \frac{(1 - \beta_z \sqrt{\varepsilon - \sin^2 \theta_z} - \beta_z^2 - \beta_x \cos \theta_x) \sin^2 \theta_z + \beta_x \beta_z \cos \theta_x \sqrt{\varepsilon - \sin^2 \theta_z}}{(1 - \beta_x \cos \theta_x - \beta_z \sqrt{\varepsilon - \sin^2 \theta_z})(\sqrt{\varepsilon - \sin^2 \theta_z} + \varepsilon \cos \theta_x)} \right|^2. \quad (1)$$

The spectral-angular density of radiation perpendicular to the radiation plane is determined for the component with polarization by formula:

$$I^{\perp}(\mathbf{n}, \omega) = \frac{\beta_x^2 \beta_z^4 \cos^2 \theta_y \cos^2 \theta_z |1 - \varepsilon|^2}{\left[(1 - \beta_x \cos \theta_x)^2 - \beta_z^2 \cos^2 \theta_z \right]^2 \sin^2 \theta_z} \times \left| \left(1 - \beta_x \cos \theta_x - \beta_z \sqrt{\varepsilon - \sin^2 \theta_z} \right) \left(\sqrt{\varepsilon - \sin^2 \theta_z} + \cos \theta_x \right) \right|^{-2}. \quad (2)$$

The direction of the electron velocity forms an angle ψ with the normal to the target surface, such that $\beta_x = \beta \sin \psi$, $\beta_z = \beta \cos \psi$, $\beta = v / c$. The direction of radiation is determined by the direction cosines relative to the coordinate axes (z -axis coincides with the normal to the surface): $\cos \theta_x = \sin \theta \cos \varphi$, $\cos \theta_y = \sin \theta \sin \varphi$, $\cos \theta_z = \cos \theta$. The dimensionless spectral-angular density of radiation intensity in formulas (1) and (2) is expressed in units of $Z^2 e^2 / \pi^2 c$ (Ze is the particle charge).

Fig. 1 shows calculations of the angular distribution of polarized radiation from electrons with an energy of 10 MeV in a titanium target of $10 \mu\text{m}$ thickness near the L -absorption edge (454 eV) at incidence angles of 50° and 70° , respectively. The vertical line marks the

direction cosines of the particle incidence angle ψ relative to the surface normal. Fig. 2 presents the results of calculations of the angular distribution of radiation at zero azimuthal angle. Two maxima corresponding to the Cherenkov radiation cone are clearly visible. As follows from the performed calculations, the intensity of the longitudinally polarized wave increases with an increasing particle incidence angle into the crystal, while the angular width of the maximum decreases. At an incidence angle into the titanium target of 70° , the radiation yield at the maximum can exceed the yield at normal incidence by approximately a factor of twenty. Given the measurement results [6], this would amount to approximately 7×10^{-3} photons per electron.

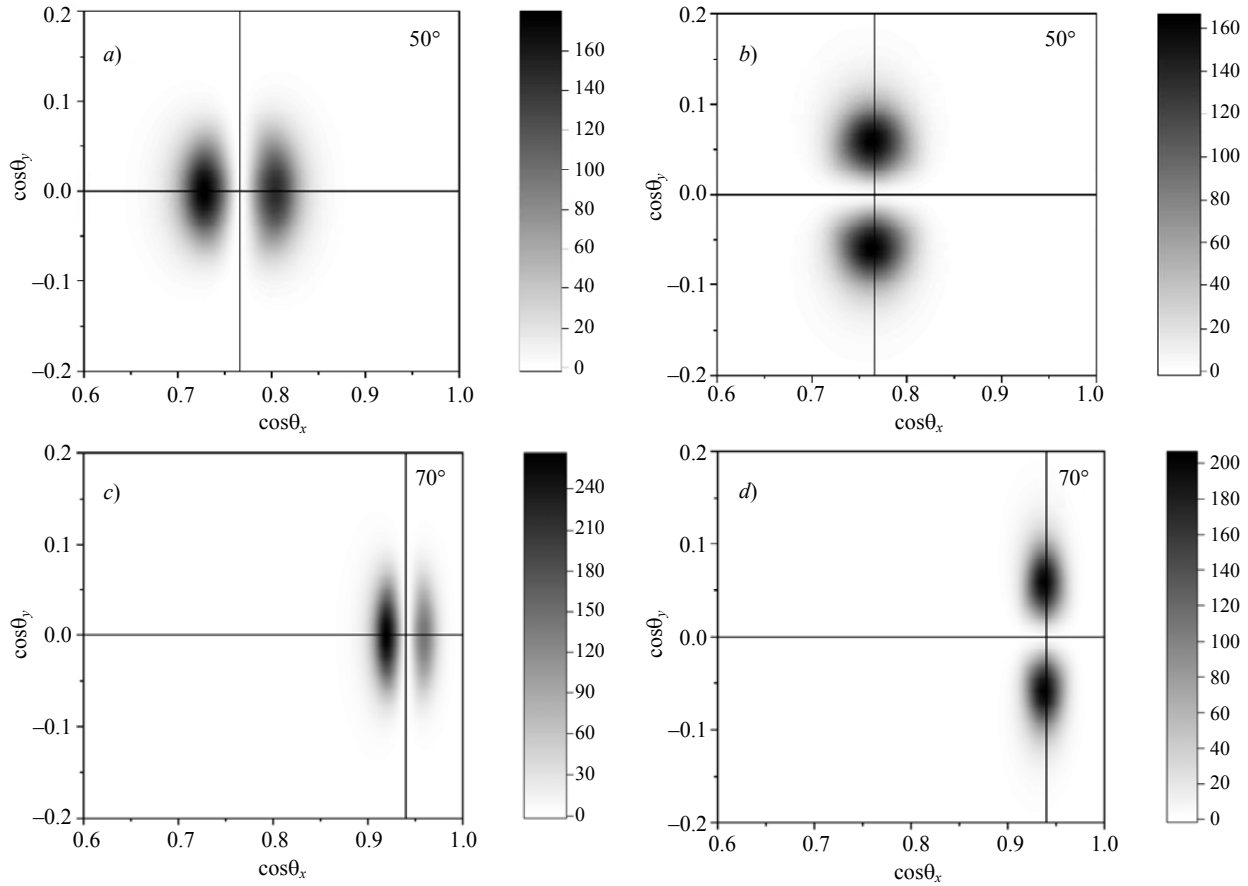


Fig. 1. Calculated angular distribution of the longitudinally polarized (a, c) and transversely polarized (b, d) components of Cherenkov radiation from relativistic electrons with the 10 MeV energy in the X-ray frequency range ($\omega = 454$ eV) in the titanium target at different particle incidence angles: (a, b) – $\psi = 50^\circ$, (c, d) – $\psi = 70^\circ$

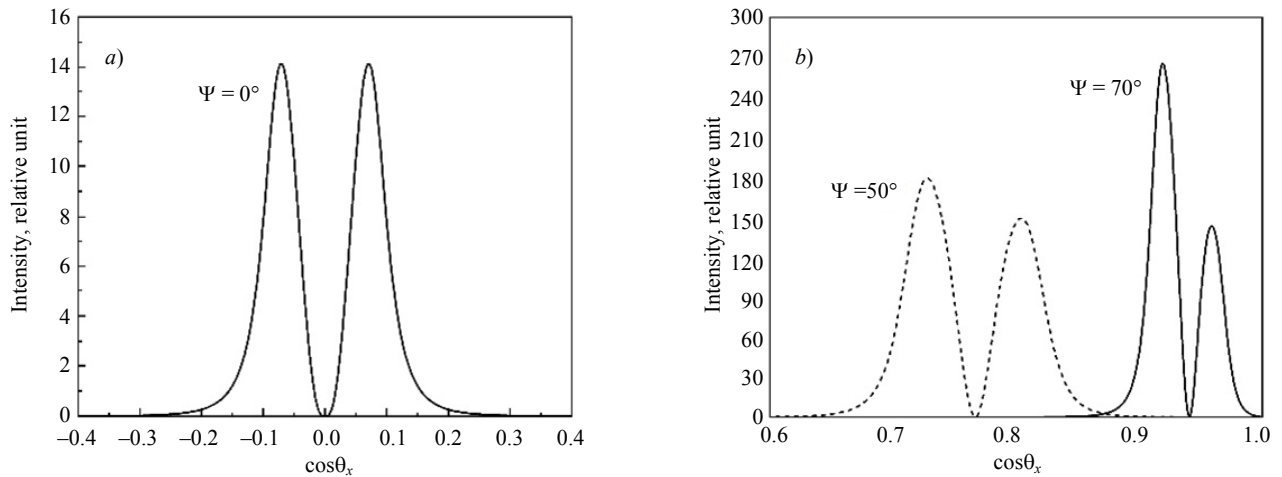


Fig. 2. Calculated angular distribution in the polar angle in the plane of Cherenkov radiation (azimuthal angle $\psi = 0$) with longitudinal polarization for relativistic electrons with the 10 MeV energy in the X-ray frequency range ($\omega = 454$ eV) in the titanium target at different particle incidence angles.

Parametric X-Ray Radiation

When relativistic electrons pass through a regular stack of plates, along with the well-known Cherenkov and transition radiation, the so-called resonant parametric (or structural) Cherenkov radiation arises. The intensity of the resonant

parametric radiation can, in some cases, exceed the radiation intensity of an isolated target by many times. Under ideal conditions with no absorption, the intensity at the parametric maxima of the angular distribution is proportional to the square of the number of plates, N , and their angular width is proportional to $1/N$. In this case,

the position of the maxima in the angular distribution can be determined from kinematic relations [10]. In particular, if the conditions $|\varepsilon - 1| \ll 1$ and $L \ll d$ are satisfied, where d is the distance between the plates, then the angular position of the parametric maxima is determined by a well-known relation $\cos \theta_n \approx 1 / \sqrt{\varepsilon' \beta + 2\pi n c / \omega d}$, where $n = \pm 1, \pm 2 \dots$. As was noted in the previous section, the existence of absorption leads to the fact that the trajectory length of the particle in the target over which coherent summation of radiation occurs becomes finite, and under the conditions chosen above, it does not exceed $0.5 \mu\text{m}$ according to estimates. If in an isolated target this results in radiation being formed in a thin layer near the rear surface, then in a regular stack of thin plates ($L < 0.5 \mu\text{m}$) this may limit the number of plates whose radiation will be summed up coherently. In this case, the position of the maxima in the angular distribution, their width, and intensity will not be determined by simple kinematic relations, but can only be found by numerical calculation of the spectral-angular characteristics of the radiation [9]. In this section, we consider the X-ray parametric radiation in the water window range from relativistic electrons with an energy of 10 MeV that pass through several identical titanium plates regularly spaced in a vacuum and placed parallel to each other.

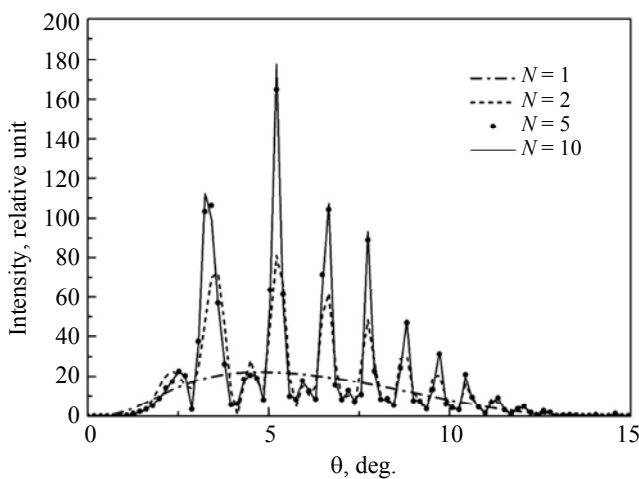


Fig. 3. The calculated angular distribution of parametric radiation from relativistic electrons with an energy of 10 MeV in the X-ray frequency range ($\omega = 454 \text{ eV}$) in a target consisting of a regular stack of N titanium plates of $0.1 \mu\text{m}$ thickness and a $1 \mu\text{m}$ vacuum gap between the plates

For calculations of the parametric X-ray radiation yield, we use the well-known result from [9], which applies recurrent relations expressing the fields in a given vacuum gap through the fields in the previous gap. The resulting exact expressions are rather cumbersome (see formula (3.1) in [9]) and will not be presented here. Fig. 3 shows the results of calculations of the angular distribution of parametric radiation from electrons at the normal incidence of 10 MeV relativistic electrons on the stack of thin titanium plates. As before, the intensity is expressed in dimensionless units of $Z^2 e^2 / \pi^2 c$. Due to the constructive interference of radiation when the particle passes through different plates, the angular distribution has a number of additional parametric maxima as compared to the analogous distribution for a single plate. Unlike the oblique incidence on an isolated target, the angular width of these radiation maxima is significantly smaller. Analysis of the calculation results shows that, as noted above, the existence of absorption leads to the fact that radiation from the regular stack of thin (no more than $0.5 \mu\text{m}$) titanium plates is formed mainly in the last few plates. Therefore, it is impossible to achieve an increase in the radiation yield by more than a factor of ten by increasing the number of plates, as shown by the calculation results in Fig. 3: namely, starting from $N = 5$, the radiation yield practically does not change.

Conclusion

The performed analysis and the established features of the Cherenkov and parametric radiation in the X-ray frequency range may be useful in the design of radiation sources and demonstrate the fundamental feasibility of using the effect to create quasi-monochromatic directional sources of polarized X-ray radiation in the spectral range of the water transparency window.

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