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Study of calculation of the induced current pulse in the measuring grids of a micrometeoroid and space debris sensor

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A formula is derived to calculate the induced current pulse excited by a grid-electrode during the charged microparticle flight. The data obtained can be used to analyze the velocity and microparticle entry angle into the grid sensor of high-speed microparticles.

Keywords: Micrometeoroids; space debris; Shockley-Ramo theorem; induced current; electrode.

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Introduction

One of the important tasks of space research is to track and characterize high-speed microparticles (micrometeoroids and space debris) [1–4]. Micron-sized particles are monitored using *in-situ* sensors, which are mounted directly on the spacecraft surface. Grid construction sensors [5, 6] are example of such sensors. They determine the velocity vector of high-speed microparticles by measuring the induced charge on the grid-electrodes in proportion to the particle's velocity and electrical charge (Figs. 1, 2).

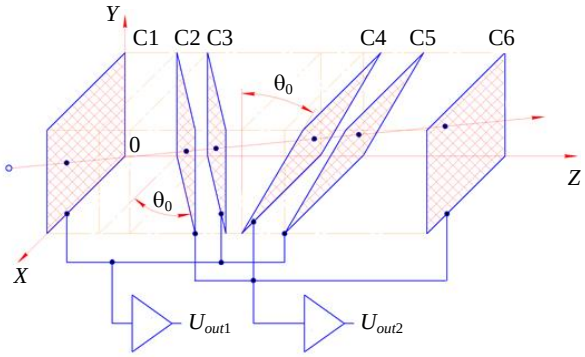

Fig. 1. Structure diagram of the grid sensor

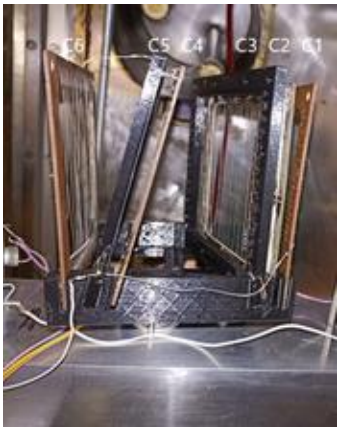

Fig. 2. Photograph of the grid construction sensor: C1, C2, C3, C4, C5, C6 – six grids are positioned as shown in Fig. 1

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The authors developed and evaluated a prototype grid sensor (Figs. 1, 2), that consists of six grid measuring electrodes arranged in tilted pairs at different angles to measure three velocity vector projections ($\vec{V} = \vec{1}_x \cdot Vx + \vec{1}_y \cdot Vy + \vec{1}_z \cdot Vz$, where $\vec{1}_x$ – unitary vector on-axis 0*X*, $\vec{1}_y$ – unitary vector on-axis 0*Y*, $\vec{1}_z$ – unitary vector on-axis 0*Z*) of a microparticle (Fig. 1). To effectively design devices based on the induced charge measurement, it is to understand the signal waveform from the measuring electrode. This knowledge is crucial for developing a method for processing the obtained experimental data [7–9]. Previous research [10–12] established a method for estimating the induced current pulse in a plane-parallel capacitor. However, in the prototype we have developed, some grids are not arranged in parallel configuration.

In this paper, we analyzed the motion of a microparticle within a planar geometry, under the condition that the particle does not fly by the grid edges.

The objective was to determine the shape of the induced current pulse waveform of non-parallel measuring grids.

Model of induced current disregarding the influence of the input measuring circuit

Figure 3 shows the projections of two non-parallel planes angled θ_0 towards each other, corresponding to the first two grids of the sensor shown in Figures 1 and 2. A charged microparticle (a micrometeoroid or space debris) flies through the first measurement grid at point M0 (*x*0,0) and flies out the second grid at point M1(*x*1, *z*1). We define the intermediate position of the microparticle at time point *t* as point M(*x*, *z*).

Given the under study microparticle's velocity is much less than the electromagnetic wave velocity, we use the steady-state approximation of the Shockley-Ramo theorem to determine the induced current pulse on the electrode [10–13]:

$$i(t) = Q \cdot \left(\vec{V} \cdot \vec{E}_k\right), \quad (1)$$

where $\vec{E}_k$ is the effective (weighted) field intensity at the point of charge *Q* at time point *t*. It's calculated by removing the charge, setting the measuring electrode's potential to 1, setting the remaining electrode potentials to 0. $\left(\vec{V} \cdot \vec{E}_k\right)$ is the dot product of the velocity vector and effective strength.

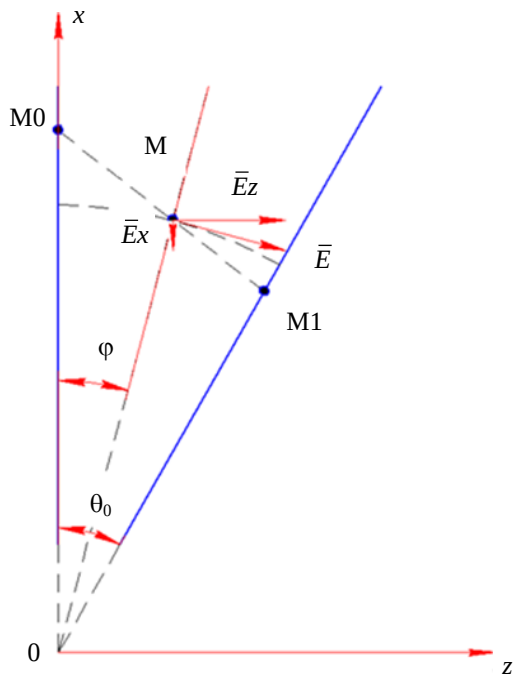

Fig. 3. Relative positions of inclined measuring grid electrodes

The electric field intensity between non-parallel plane electrodes can be calculated using the following formula in polar coordinate system [14]:

$$E = \frac{U}{r \cdot \theta_0}, \quad (2)$$

where *U* is the voltage across the measuring electrode (assumed to be 1V), *r* is the distance from point 0 to point M, and θ_0 – is the angle between the two grid electrode planes.

We will assume that the first grid C1 is at 0 potential, and the signal is measured from the second grid C2.

The coordinates of a microparticle in the Cartesian coordinate system are calculated using formulas:

$$z = r \cdot \sin(\varphi), \quad x = r \cdot \cos(\varphi), \tag{3}$$

$$x1 = x0 - Vx \cdot t, \quad z1 = Vz \cdot t. \tag{4}$$

Given $tg(\alpha) = \frac{Vx}{Vz}$

the intensity vector projections Ex, Ez in the Cartesian coordinate system are calculated using formulas:

$$Ex = E \cdot \sin(\varphi), \quad Ez = E \cdot \cos(\varphi),$$
$$tg(\varphi) = \frac{z}{x} = \frac{Vz \cdot t}{x0 - Vx \cdot t} = \frac{t}{x0/Vz - tg(\alpha) \cdot t}. \tag{5}$$

The induced current according to the Shockley-Ramo theorem is calculated using formulas:

$$\begin{aligned} i &= Q \cdot \left(\vec{V} \cdot \vec{E}\right) = .Q \cdot \left(\vec{V}x \cdot \vec{E}x + \vec{V}z \cdot \vec{E}z\right) = \\ &= \frac{U \cdot Q}{z \cdot \theta_0}\left(Vx \cdot \sin^2(\varphi) + Vz \cdot \cos(\varphi) \cdot \sin(\varphi)\right) = \\ &= \frac{U \cdot Q}{\theta_0}\left(\frac{x0/Vz}{\left(x0/Vz - tg(\alpha) \cdot t\right)^2 + t^2}\right) = \\ &= i(t=0) \cdot \left(\frac{x0^2}{\left(x0 - tg(\alpha) \cdot t \cdot Vz\right)^2 + \left(t \cdot Vz\right)^2}\right), \end{aligned} \tag{6}$$

where $i(t=0) = \frac{U \cdot Q}{\theta_0}\left(\frac{Vz}{x0}\right) = \frac{1 \cdot Q}{\theta_0}\left(\frac{Vz}{x0}\right)$ is the current at time point when the microparticle flies by the point M0.

The time of flight of the second grid C2 (at point M1) is determined by the solution of the of equation:

$$\begin{cases} x1 = x0 - Vx \cdot t_1, \\ z1 = Vz \cdot t_1, \\ tg(\theta_0) = \frac{z1}{x1}. \end{cases} \tag{7}$$

The solution of the of equations presented is expressed as:

$$t_1 = \frac{x0}{Vz \cdot \left(\frac{1}{tg(\theta_0)} + tg(\alpha)\right)} \tag{8}$$

where t_1 is the microparticle time of flight of the second grid C2.

The velocity vector component Vz can be determined by measuring the microparticle time of flight of C1 and C6 grids (Fig. 1) [5]. The velocity vector component Vx can be calculated using formula:

$$Vx = \frac{x0}{t_1} - \frac{Vz}{tg(\theta_0)}. \tag{9}$$

Then the induced current at the microparticle time of flight of the second grid C2 can be calculated using formula:

$$i\left(t=t_1\right)=\frac{U\cdot Q}{\theta_0}\left(\frac{x0/Vz}{\left(x0/Vz-tg\left(\alpha\right)\cdot\frac{x0/Vz}{\left(\frac{1}{tg\left(\theta_0\right)}+tg\left(\alpha\right)\right)}\right)^2+\left(\frac{x0/Vz}{\left(\frac{1}{tg\left(\theta_0\right)}+tg\left(\alpha\right)\right)}\right)^2}\right)= \tag{10}$$

$$=i\left(t=0\right)\cdot\left(\frac{\left(ctg\left(\theta_0\right)+tg\left(\alpha\right)\right)^2}{ctg^2\left(\theta_0\right)+1}\right).$$

The entry angle of a microparticle into the first grid C1 can be calculated using formula:

$$tg\left(\alpha\right)=\sqrt{i\left(t=t1\right)\cdot\left(ctg^2\left(\theta_0\right)+1\right)/i\left(t=0\right)}- \\ -ctg\left(\theta_0\right). \tag{11}$$

At $\alpha=0\left(tg\left(\alpha\right)=0\right)$ we obtain a formula for the induced current at the moment of the microparticle passing the grid C2:

$$i\left(t=t_1\right)=i\left(t=0\right)\cdot\left(\frac{1}{1+tg^2\left(\theta_0\right)}\right)= \\ =i\left(t=0\right)\cdot\cos^2\left(\theta_0\right). \tag{12}$$

Given the microparticle's charge Q and velocity Vz, we can calculate the coordinate $x0$ of its passage through grid C1:

$$x0=\frac{U\cdot Q}{\theta_0}\left(\frac{Vz}{i\left(t=0\right)}\right). \tag{13}$$

Model of induced current considering the influence of the input measuring circuit

High-speed microparticle sensors shall consider the impact of input measurement circuit parameters like input capacity, amplifier resistance, and wiring parasitic inductance [12]. The differential equation derived from the radiotechnical model needs to be solved (Fig. 4) [10, 15]:

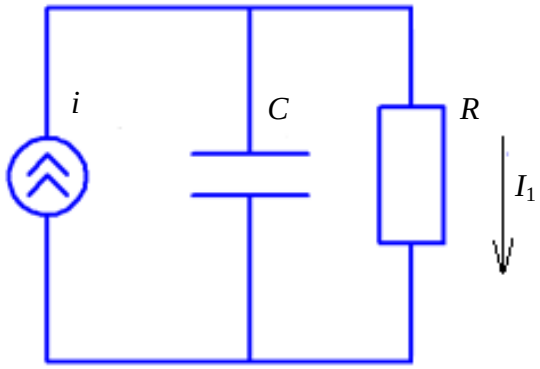

Fig. 4. A radiotechnical model of the input circuit: C – input capacity; R – input resistance

By solving the differential equation, we can determine the expression that describes the current I_1, flowing through the input resistance [10, 12]:

$$C\cdot R\cdot\frac{dI_1}{dt}+I_1=i. \tag{14}$$

To simplify the analytical solution of this differential equation, we will expand the induced current formula using Taylor's theorem and limit it to the first three terms of

the series (the error of such approximation shall not exceed 0.5 %):

$$i = i(t=0)\times$$
$$\times\left(1+\frac{2\cdot Vx}{x0}\cdot t+\left(3\cdot\left(\frac{Vx}{x0}\right)^2-\left(\frac{Vz}{x0}\right)^2\right)\cdot t^2\right)= \quad (15)$$
$$= i(t=0)\cdot\left(1+A_1\cdot t+A_2\cdot t^2\right),$$

where $A_1=\frac{2\cdot Vx}{x0}$, $A_2=\left(3\cdot\left(\frac{Vx}{x0}\right)^2-\left(\frac{Vz}{x0}\right)^2\right)$.

We will apply the initial time point during which the microparticle passes the first grid (C1), $I_1(t=0)=0$, when solving the differential equation. Therefore, the formula for the current flowing through the resistor is:

$$I_1 = i(t=0)\times$$
$$\times\left[\begin{array}{l}\left(1-A_1\cdot\tau+2\cdot A_2\cdot\tau^2\right)\cdot\left(1-e^{-t/\tau}\right)+\\+\left(A_1-2\cdot\tau\cdot A_2\right)\cdot t+A_2\cdot t^2\end{array}\right], \quad (16)$$

where $\tau = R\cdot C$ is the input time constant.

At $\tau\to 0$ the formula changes to a formula disregarding the influence of the input measuring circuit. The presented formula can also be expanded using a Taylor's theorem, but only at $t/\tau<<1$, otherwise a sufficient number of terms of the series shall be included for accuracy.

Figs. 5, 6 show the dependency diagrams of current change in time. For convenience of analysis, we will plot the graph of the specified current and time functions coordinates:

$$I_{NORM1} = I_1 / i(t=0) =$$
$$=\left[\begin{array}{l}\left(1-A_1\cdot\tau+2\cdot A_2\cdot\tau^2\right)\cdot\left(1-e^{-t/\tau}\right)+\\+\left(A_1-2\cdot\tau\cdot A_2\right)\cdot t+A_2\cdot t^2\end{array}\right], \quad (17)$$

$$t_{norm} = t/t_1. \quad (18)$$

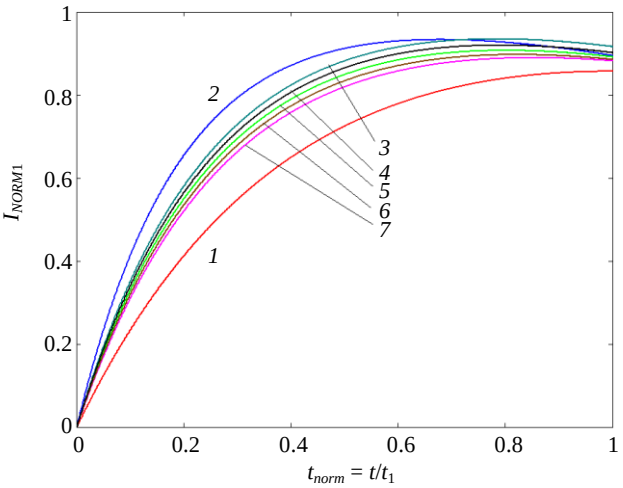

Fig. 5. The specified current pulse graph between the first and second grid at Vz = 1000 m/s, $\tau = 10^{-5}$ s: 1 – x0 = 0.1 m, α = 0; 2 – x0 = 0.2 m, α = 0; 3 – x0 = 0.15 m, α = -10°; 4 – x0 = 0.15 m, α = -5°; 5 – x0 = 0.15 m, α = 0°; 6 – x0 = 0.15 m, α = 5°; 7 – x0 = = 0.15 m, α = 10°

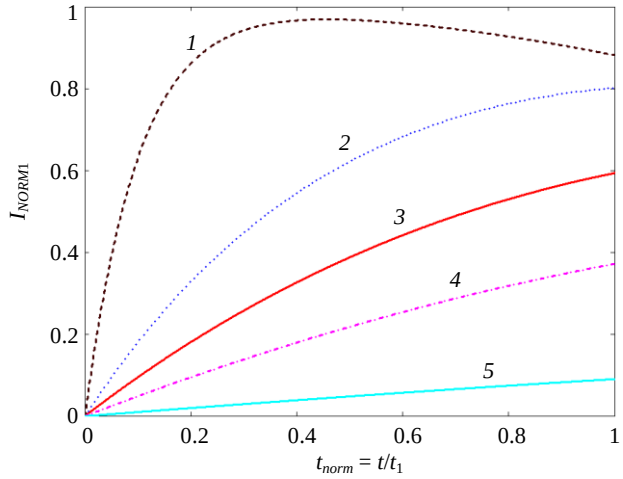

Fig. 6. The specified current pulse graph between the first and second grid at x0 = 0.15 m, α = 0°, Vz = 1000 m/s: 1 – t_1/τ = 10; 2 – t_1/τ = 2; 3 – t_1/τ = 1; 4 – t_1/τ = 0,5; 5 – t_1/τ = 0,1

Conclusion

We propose an analytical model to describe the induced current in a sensor with non-parallel measuring grids, designed to measure the velocity vector of micrometeoroids and space debris. A large

input circuit time constant causes a decrease in the input current amplitude, which can lead to a decrease in the sensor's ability to accurately measure fast microparticles. On the other hand, the discharge of the input capacity causes a drop in current amplitude when measuring slow microparticles. It was noted that the angle at which the microparticle enters the first grid also influences the measured signal waveform. The obtained data can be used in selecting the geometrical dimensions of a high-speed microparticle sensor.

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